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ScienceDirect
Procedia Engineering 127 (2015) 916 – 923
Procedia Engineering
www.elsevier.com/locate/procedia

International Conference on Computational Heat and Mass Transfer-2015

Effect of radiation on MHD convective flow and heat transfer of a viscoelastic fluid over a stretching surface

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Abstract

The Problem of three-dimensional MHD convective boundary layer flow and heat transfer of a viscoelastic fluid in the presence of radiation was examined. A similarity transformation is used to reduce the governing non-linear partial differential equations into ordinary differential equations and then they are solved using homotopy analysis method (HAM). Graphical results for the velocity and temperature profiles are displayed and discussed. Numerical results for the local Nusselt number is tabulated for various values of the parameters.

Peer-review under responsibility of the organizing committee of ICCHMT – 2015

Keywords: Viscoelastic fluid, Heat transfer, MHD, Radiation, HAM

1. Introduction

In recent years, considerable interest has been evinced in the study of boundary layer flow and heat transfer of a viscoelastic fluid over a stretching surface, because of their importance in many industrial applications like polymer sheet extrusion from a dye, drawing of plastic films, glass fiber, paper production, hot rolling, crystal growing and production of synthetic sheets. Hayat et al. [1] studied the three-dimensional flow of an incompressible elastico-viscous fluid over a stretching surface. Hayat et al. [2] analyzed the three-dimensional flow and mass transfer of a viscoelastic fluid over a stretching sheet. Three-dimensional magneto-hydrodynamic flow and heat transfer of a

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Peer-review under responsibility of the organizing committee of ICCHMT – 2015
doi:10.1016/j.proeng.2015.11.364

viscoelastic fluid over a stretching/shrinking surface was studied by Turkyilmazoglu [3].

Magneto-hydrodynamics (MHD) was virtually applied in MHD generators, pumps, accelerators, flow meters and blood flow measurements, etc. Free convection boundary layer flow of a viscoelastic fluid past an infinite vertical porous plate under the influence of magnetic field was analytically studied by Chowdhury and Islam [4]. The effects of radiation and chemical reaction on MHD boundary layer flow over an exponentially stretching sheet were investigated by Seini and Makinde [5]. The influence of radiation on unsteady MHD flow and heat transfer past a semi-infinite vertical porous plate was analyzed by Karthikeyan et al. [6]. The effect of thermal radiation on MHD boundary layer flow and heat transfer of a viscoelastic fluid over a stretching sheet was numerically analyzed by Abel and Mahesha [7]. They found that the temperature and thermal boundary layer thickness increases on increasing the magnetic field parameter. MHD free convection flow and heat transfer of a viscous fluid along a vertical flat plate in the presence of heat generation was studied by Safiqul Islam et al. [8]. Bhuvaneswari et al. [9] examined the heat transfer and fluid flow of an incompressible viscous fluid along a semi-infinite inclined surface in a fluid-saturated heat generating porous medium. MHD free convection flow and heat transfer of a viscous fluid with temperature dependent thermal conductivity and heat absorption over a vertical wavy surface was numerically analyzed by Parveen and Abdul Ali [10].

The objective of the current investigation is to study the unsteady MHD three-dimensional flow and heat transfer of a viscoelastic fluid over a stretching surface in the presence of radiation using homotopy analysis method.

2. Mathematical formulation

Let us consider an unsteady three-dimensional boundary layer flow of a viscoelastic fluid over a stretching surface. It is assumed that the surface is stretched with velocity $u_w(x)$, along x −axis keeping the origin is fixed and y −axis is perpendicular to it. A consistent magnetic field of strength B_0 is applied in the $y-$ direction. It is assumed that the magnetic Reynolds number is very small, so the induced magnetic field can be neglected. The surface is kept at a constant temperature T_w which is higher than the surrounding fluid temperature T_∞. The unsteady MHD convective flow is governed by the following equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\nu\frac{\partial^2 u}{\partial z^2}-k_0\begin{pmatrix}\frac{\partial^3 u}{\partial z^2\partial t}+u\frac{\partial^3 u}{\partial x\partial z^2}+w\frac{\partial^3 u}{\partial z^3}-\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial z^2}\\ -\frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial z^2}-2\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial x\partial z}-2\frac{\partial w}{\partial z}\frac{\partial^2 u}{\partial z^2}\end{pmatrix}-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=\nu\frac{\partial^2 v}{\partial z^2}-k_0\begin{pmatrix}\frac{\partial^3 v}{\partial z^2\partial t}+v\frac{\partial^3 v}{\partial x\partial z^2}+w\frac{\partial^3 v}{\partial z^3}-\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial z^2}\\ -\frac{\partial v}{\partial z}\frac{\partial^2 w}{\partial z^2}-2\frac{\partial v}{\partial z}\frac{\partial^2 u}{\partial y\partial z}-2\frac{\partial w}{\partial z}\frac{\partial^2 v}{\partial z^2}\end{pmatrix}-\frac{\sigma B_0^2}{\rho}v \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha_m\frac{\partial^2 T}{\partial z^2}+\frac{Q}{\rho c_\rho}(T-T_\infty)-\frac{\alpha_m}{k_T}\frac{\partial q_r}{\partial z} \tag{4}$$

where u, v and w are the velocity components, x, y and z are the space coordinates, ν is the kinematic viscosity, k_0 is the material fluid parameter, ρ is the density of the fluid, α_m is the thermal diffusivity, Q is the internal heat generation (>0) or absorption (<0) of the fluid, c_p is the specific heat and k_T is the thermal diffusion ratio.

Using Rosseland approximation, the radiative heat flux is taken as $q_r=-\frac{\sigma_0}{3k^*}\frac{\partial T^4}{\partial z}$, where σ_0 and k^* are the Stefan-Boltzmann constant and mean absorption coefficient. Using Taylor series, we get $T^4 \cong 4T_\infty^3 T-3T_\infty^4$.

The appropriate boundary conditions are,

$$u=u_w(x)=\frac{ax}{1-\alpha t},\ \ v=u_w(y)=\frac{by}{1-\alpha t}, w=0, T=T_{w,}\ \ at\ z=0$$

$$u \to 0, v \to 0, \frac{\partial u}{\partial z} \to 0, \frac{\partial v}{\partial z} \to 0, T \to T_\infty, \quad as\ z \to \infty \tag{5}$$

where a and b are positive constants.

The following transformations are used to find the non-dimensional equations of the problem:

$$\eta = \sqrt{\frac{a}{v(1-\alpha t)}}z, u = \frac{ax}{1-\alpha t}f'(\eta), v = \frac{ay}{1-\alpha t}g'(\eta), w = -\sqrt{\frac{av}{(1-\alpha t)}}\left(f(\eta)+g(\eta)\right), \theta = \frac{T-T_\infty}{T_w - T_\infty} \tag{6}$$

Using (6), Equations (2)-(4) can be reduced as

$$\begin{aligned} &f''' - f'^2 - \zeta\left(\frac{\eta}{2}f'' + f'\right) + (f+g)f'' \\ &\quad + K\left[-\zeta\left(\frac{\eta}{2}f^{iv} + 2f'''\right) + (f+g)f^{iv} + (f''-g'')f'' - 2(f'+g')f'''\right] - Haf' = 0 \end{aligned} \tag{7}$$

$$\begin{aligned} &g''' - g'^2 - \zeta\left(\frac{\eta}{2}g'' + g'\right) + (f+g)g'' \\ &\quad + K\left[-\zeta\left(\frac{\eta}{2}g^{iv} + 2g'''\right) + (f+g)g^{iv} - (f''-g'')g'' - 2(f'+g')g'''\right] - Hag' = 0 \end{aligned} \tag{8}$$

$$\left(1 + \frac{4}{3R_d}\right)\theta'' + Pr(f+g)\theta' - Pr\frac{\zeta}{2}\eta\theta' + PrH_g\theta = 0 \tag{9}$$

Boundary conditions (5) interms of f, g and θ becomes,

$$f(0) = 0, f'(0) = 1, f'(\infty) = 0, g(0) = 0,\ g'(0) = c,\ g'(\infty) = 0, \theta(0) = 1, \theta(\infty) = 0 \tag{10}$$

where $c = \frac{b}{a}$ is the stretching ratio, $Ha = \frac{\sigma B_0^2(1-\alpha t)}{\rho a}$ is the Hartmann number, $H_g = \frac{Q(1-\alpha t)}{a\rho c_\rho}$ is the internal heat generation/absorption parameter, $K = \frac{k_0 a}{v(1-\alpha t)}$ is the viscoelastic parameter, $Pr = \frac{v}{\alpha_m}$ is the Prandtl number, $R_d = \frac{k^* k_T}{4\sigma_0 T_\infty^3}$ is the radiation parameter, $\zeta = \frac{\alpha_m}{a}$ is the unsteady parameter.

The heat transfer rate is an important quantity in engineering applications. So, the local Nusselt number is defined by $Nu_x Re_x^{-\frac{1}{2}} = -\left(1 + \frac{4}{3R_d}\right)\theta'(0)$.

Equations $(7) - (9)$ with the boundary conditions (10) are highly non-linear ordinary differential equations. The equations are solved by homotopy analysis method.

3. Series Solutions

The initial approximations for homotopy analysis solutions are chosen as

$$f_0(\eta) = 1 - e^{-\eta};\ \ g_0(\eta) = c(1 - e^{-\eta});\ \ \theta_0(\eta) = e^{-\eta};$$

the auxiliary linear operators L_f, L_θ and L_ϕ as

$$L_f = f''' - f';\quad L_g = g''' - g';\quad L_\theta = \theta'' - \theta,$$

with satisfying the following properties

$$L_f[C_1 + C_2e^\eta + C_3e^{-\eta}] = 0;\ \ L_g[C_4 + C_5e^\eta + C_6e^{-\eta}] = 0;\ \ L_\theta[C_7e^\eta + C_8e^{-\eta}] = 0;$$

where $C_i, (i = 1 - 8)$ denote the arbitrary constants.

The zero^th order deformation problems are

$$(1-p)L_f\left[\bar{f}(\eta,p) - f_0(\eta)\right] = ph_f N_f\left[\bar{f}(\eta,p), \bar{g}(\eta,p)\right],$$

$$(1-p)L_g\left[\bar{g}(\eta,p) - g_0(\eta)\right] = ph_g N_f\left[\bar{f}(\eta,p), \bar{g}(\eta,p)\right],$$

$$(1-p)L_\theta\left[\bar{\theta}(\eta,p) - \theta_0(\eta)\right] = ph_\theta N_\theta\left[\bar{f}(\eta,p), \bar{g}(\eta,p), \bar{\theta}(\eta,p)\right]$$

$$\bar{f}(0,p) = 0, \bar{f}'(0,p) = 1, \bar{f}'(\infty,p) = 0, \bar{g}(0,p) = 0,\ \bar{g}'(0,p) = c, \bar{g}'(\infty,p) = 0, \bar{\theta}(0,p) = 1, \bar{\theta}(\infty,p) = 0.$$

The m^{th}-order deformation problem is of the form

$$L_f[f_m(\eta) - \chi_m f_{m-1}(\eta)] = h_f R_m^f(\eta) \quad (11)$$

$$L_g[g_m(\eta) - \chi_m g_{m-1}(\eta)] = h_g R_m^g(\eta) \quad (12)$$

$$L_\theta[\theta_m(\eta) - \chi_m \theta_{m-1}(\eta)] = h_\theta R_m^\theta(\eta) \quad (13)$$

$$f_m(0) = 0, {f_m}'(0) = 0, g_m(0) = 0, {g_m}'(0) = 0,\ \theta_m(0) = 0, {f_m}'(\infty) = 0, {g_m}'(\infty) = 0, \theta_m(\infty) = 0$$

where

$$\begin{aligned} R_m^f(\eta) = {} & f'''_{m-1}(\eta) - \zeta\left(\frac{\eta}{2}f''_{m-1}(\eta) + f'_{m-1}(\eta)\right) - K\,\zeta\left(\frac{\eta}{2}f_{m-1}^{iv}(\eta) + 2f'''_{m-1}(\eta)\right) \\ & + \sum_{k=0}^{m-1}\Big\{\big(f_{m-1-k}(\eta) + g_{m-1-k}(\eta)\big)f_k''(\eta) - f'_{m-1-k}(\eta)f_k'(\eta) \\ & + K\Big[\big(f_{m-1-k}(\eta) + g_{m-1-k}(\eta)\big)f_k^{iv} + \big(f''_{m-1-k}(\eta) - g''_{m-1-k}(\eta)\big)f_k''(\eta) \\ & - 2\big(f'_{m-1-k}(\eta) + g'_{m-1-k}(\eta)\big)f_k'''(\eta)\Big]\Big\} \end{aligned}$$

$$\begin{aligned} R_m^g(\eta) = {} & g'''_{m-1}(\eta) - \zeta\left(\frac{\eta}{2}g''_{m-1}(\eta) + g'_{m-1}(\eta)\right) - K\,\zeta\left(\frac{\eta}{2}g_{m-1}^{iv}(\eta) + 2g'''_{m-1}(\eta)\right) \\ & + \sum_{k=0}^{m-1}\Big\{\big(f_{m-1-k}(\eta) + g_{m-1-k}(\eta)\big)g_k''(\eta) - g'_{m-1-k}(\eta)g_k'(\eta) \\ & + K\Big[\big(f_{m-1-k}(\eta) + g_{m-1-k}(\eta)\big)g_k^{iv} - \big(f''_{m-1-k}(\eta) - g''_{m-1-k}(\eta)\big)g_k''(\eta) \\ & - 2\big(f'_{m-1-k}(\eta) + g'_{m-1-k}(\eta)\big)g_k'''(\eta)\Big]\Big\} \end{aligned}$$

$$R_m^\theta(\eta) = \left(1 + \frac{4}{3R_d}\right)\theta''_{m-1}(\eta) + Pr\sum_{k=0}^{m-1}\big(f_{m-1-k}(\eta) + g_{m-1-k}(\eta)\big)\,\theta_k'(\eta) - Pr\frac{\zeta}{2}\eta\theta_{m-1}'(\eta) + PrH_g\theta_{m-1}(\eta)$$

and

$$\chi_m = \begin{cases} 0, & m \leq 1 \\ 1, & m > 1 \end{cases}$$

The general solution of Equations (11)-(13) is

$$f_m(\eta) = {f^*}_m(\eta) + C_1 + C_2e^{\eta} + C_3e^{-\eta}$$
$$g_m(\eta) = {g^*}_m(\eta) + C_4 + C_5e^{\eta} + C_6e^{-\eta}$$

$\theta_m(\eta) = \theta^*{}_m(\eta) + C_7 e^{\eta} + C_8 e^{-\eta}$

where $f^*{}_m(\eta)$, $g^*{}_m(\eta)$ and $\theta^*{}_m(\eta)$ are the special solutions. The symbolic calculations are obtained by MATLAB.

4. Results and discussion

Table 1 provides the comparison of the present study for different c values with Hayat et al. [11]. It is seen from the table that our results are agreed well with Hayat et al. [11]. Table 2 shows the variations of $-f''(0)$ and $-g''(0)$ for different values of ζ, c, K and Ha. It is seen that $-f''(0)$ and $-g''(0)$ increase on increasing the values of $\zeta, c\ K$ and Ha. Table 3 provides the values of local Nusselt number for various parameters involved in the study with $Pr = 1.0$. It is found that the surface heat transfer rate decreases on increasing the values of Ha, R_d and H_g.

Figure 1 indicates that the respective admissible value of h_f, h_g and h_θ are $-1.4 \le h_f, h_g \le -0.4$ and $-1.4 \le h_\theta \le -0.2$. We choose the values of auxiliary parameter ($h = -0.9$) from this range, we will get the more accurate results. Figures $2-3$ show the effect of unsteady parameter (ζ) on velocity profiles f' and g'. It is observed that the both x −component and y −component velocities decrease on increasing the unsteady parameter. This inhibits the growth of change of laminar to turbulent flow. The effects of viscoelastic parameter (K) on velocity profies are displayed in Figures $4-5$. It is observed that the increase in viscoelastic parameter brings a decrease in the fluid velocity. The influence of stretching ratio (c) on velocity profiles are displayed in Figures 6– 7. It is seen from these figures that the $x-$ component velocity decreases and $y-$ component velocity increases on increasing the stretching ratio. It is obvious that the stretching ratio is ratio of the coefficient of both directional velocities. Either $x-$ component velocity or $y-$ component velocity boosted up depending on the values of c. It is observed from the Figures 8 – 9 that both velocities decrease on increasing the Hartmann number (Ha). The magnetic field rises to a resistive type force called Lorentz force and this force has the tendency to slow down the motion of the fluid. The fluid velocity and their boundary layer thickness decreases with increasing the Hartmann number. Figure 10 shows the effect of viscoelastic parameter on temperature. It is observed that the fluid temperature is an increasing function of Kdue to reducing the viscosity of the fluid on increasing the values of K. Figure 11 depicts the effects of radiation on the temperature. Thermal radiation affects the rate of energy transport to the fluid and so the fluid temperature decreases on increasing the thermal radiation parameter. The effect of internal heat generation/absorption parameter (H_g) on temperature is shown in Figure 12. The effect of internal heat generation ($H_g > 0$) is to increase the rate of heat transport to the fluid thereby increasing the temperature of the fluid. However heat absorption ($H_g < 0$) is to decrease the rate of heat transport to the fluid thereby decrease the temperature of the fluid.

5. Conclusion

The problem of unsteady three-dimensional MHD boundary layer flow and heat transfer of a viscoelastic fluid over a stretching surface in the presence of radiation is considered. Analytical solutions are derived using homotopy analysis method. It is found that the momentum boundary layer thickness decreases on increasing the unsteadiness parameter, viscoelastic parameter and Hartmann number. The thermal boundary layer thickness increases on increasing the viscoelastic parameter and internal heat generation/absorption parameter. The surface heat transfer rate decreases on increasing the Hartmann number, radiation parameter and internal heat generation/absorption parameter.

Table 1: Comparison of $-f''(0)$ and $-g''(0)$ for different values of c with Hayat et al. [11]

	$-f''(0)$		$-g''(0)$	
c	Present Study	Hayat et al. (2012)	Present Study	Hayat et al. (2012)
0.0	1.000000	1.000000	0.000000	0.000000
0.3	1.057955	1.057954	0.243360	0.243359
0.5	1.093095	1.093095	0.465205	0.465204
0.8	1.142489	1.142488	0.866683	0.866682
1.0	1.173722	1.173720	1.173722	1.173720

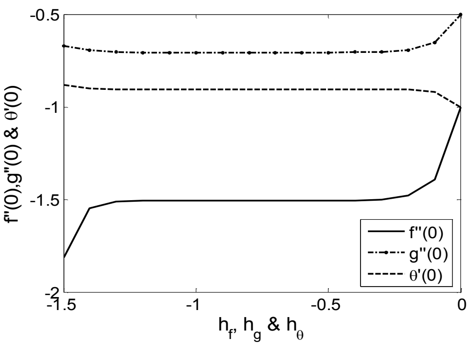

Fig. 1. $h$curves of $f''(0)$, $g''(0)$ and $\theta'(0)$.

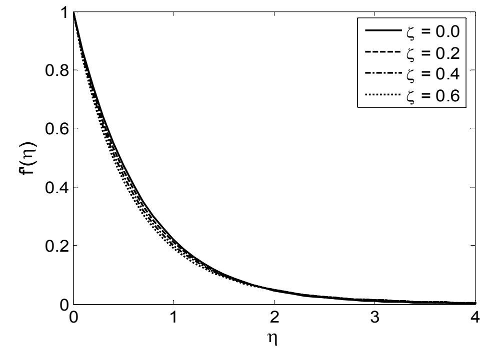

Fig. 2. Influence of ζ on $f'(\eta)$.

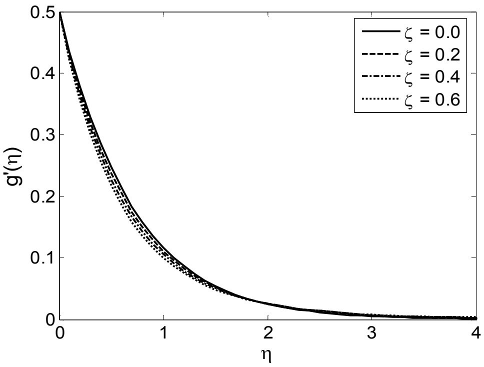

Fig. 3. Influence of ζ on $g'(\eta)$.

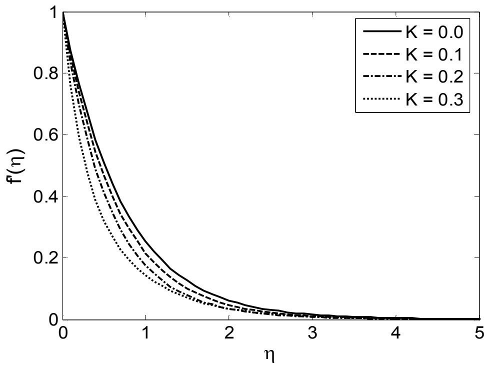

Fig. 4. Influence of K on $f'(\eta)$.

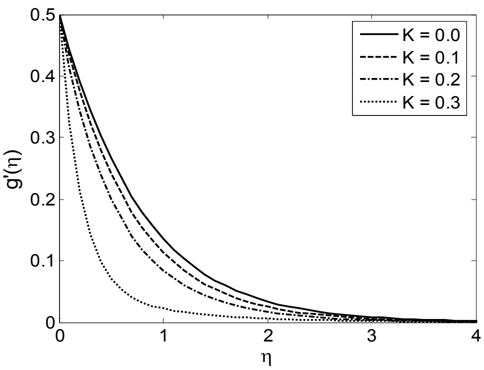

Fig. 5. Influence of K on $g'(\eta)$.

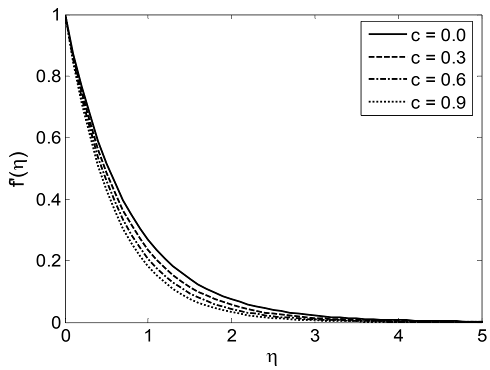

Fig. 6. Influence of c on $f'(\eta)$.

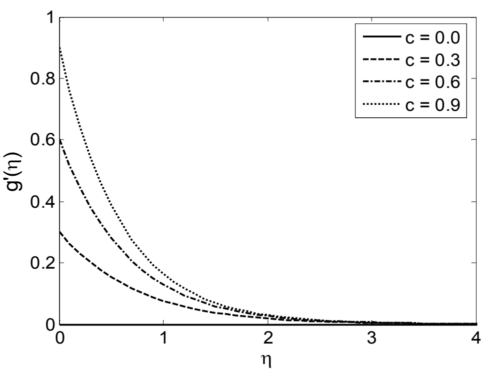

Fig. 7. Influence of c on $g'(\eta)$.

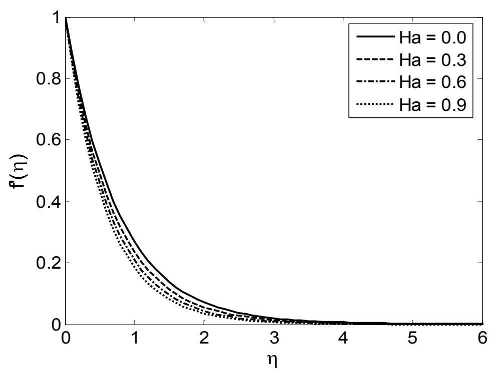

Fig. 8. Influence of Ha on $f'(\eta)$.

Table 2: Variations of $-f''(0)$ and $-g''(0)$ for different values of ζ, c, K and Ha

ζ	c	K	Ha	$-f''(0)$	$-g''(0)$
0.0	0.5	0.1	0.5	1.457281	0.679720
0.1				1.506761	0.706782
0.2				1.557360	0.734476
0.3				1.609138	0.762835
0.1	0.0	0.1	0.5	1.336235	0.000000
	0.5			1.506761	0.706783
	1.0			1.709284	1.709284
	1.5			1.971073	3.002071
0.1	0.5	0.0	0.5	1.324748	0.596271
		0.1		1.506761	0.706782
		0.2		1.827298	0.952989
		0.3		2.877361	2.206140
0.1	0.5	0.1	0.0	1.272796	0.566603
			0.3	1.417614	0.654189
			0.6	1.549473	0.731702
			0.9	1.671262	0.801914

Table 3: Variations of local Nusselt number for different values of Ha, R_d and H_g.

Ha	R_d	H_g	$Nu_x Re_x^{-\frac{1}{2}}$
0.0	10	-0.5	1.053240
0.3			1.036834
0.6			1.023073
0.9			1.011279
0.5	2	-0.5	1.209026
	5		1.076941
	10		1.027414
	15		1.010129
	20		1.001327
0.5	10	-0.5	1.027414
		-0.3	0.896717
		0.0	0.630487
		0.3	-0.027190
		0.5	-1.828167

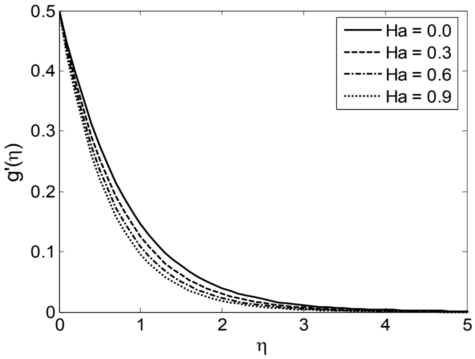

Fig. 9. Influence of Ha on $g'(\eta)$.

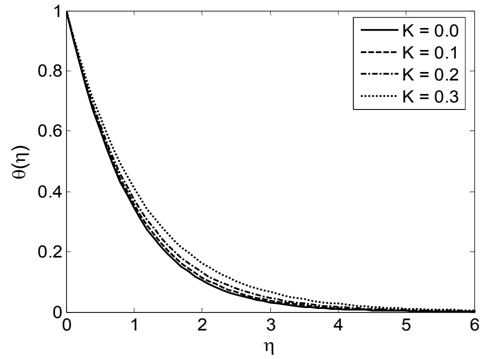

Fig. 10. Influence of K on $\theta(\eta)$.

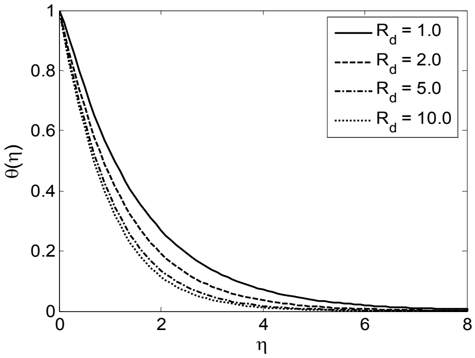

Fig. 11. Influence of R_d on $\theta(\eta)$.

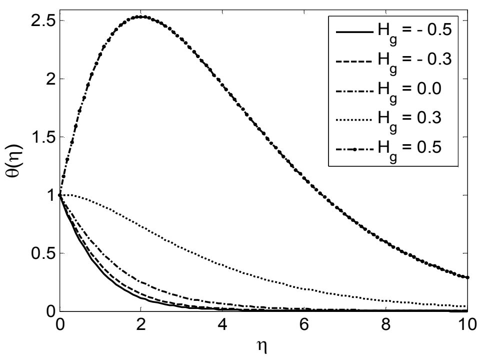

Fig. 12. Influence of H_g on $\theta(\eta)$.

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